Ph.D. Synopsis /work plan

On Topic

Some Problems in Delay Integro differential Equations

By

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1) Introduction:

Delay differential equations, neutral differential equations, integro differential equations, functional differential equations and integral equations have been studied for last many years. In the early nineteenth century, Volterra discussed the integro differential equations that model viscoelasticity in 1931, he wrote fundamental book on the role of hereditary effects on models for the interaction of species. The subject gained much momentum after1940 due to the consideration of meaningful models of engineering system and control. It is probably true that most engineers were well aware of the fact that hereditary effects occur in physical systems, but this effects was often ignored because there was insufficient theory to discuss such models in detail .During last 60 years, the theory of functional differential equations has been developed extensively and has become part of the vocabulary of researchers dealing with specific applications such as viscoelasticity, mechanics, nuclear reactors, distribute networks, heat flows, neural networks, ebidemiolody, microbiology, interaction of species, as well as many others During 1950's there was consider able activity in the subjects which led to important contributions by Myshkis [1], V.Kolmnovski and Myshkis [2], Krasovskii [3], Bellman and Cooke [4],Halanay [5]. Mathematical modles with delay take into account the memory (hereditary effects) of the dynamic system when a sequence of its past states impacts its future evolution. Most interesting applications of the Volterra integral equations are in hereditary mechanics which is also known as mechanics of materials with memory. In 1900s, Volterra was led to equations with infinite retardation (delay). Realizing that the mathematical apparatus was not yet developed at that time for undertaking successfully such investigations, he cut the infinite delay and transformed his equations into equations with finite delay. He studied the population growth where new type of equations have been developed and was termed as integro-differential equations. In this type of equations, the unknown function occurs in one the side as an ordinary derivative and appears on the other side under the integral sign. Several physics and biology problems give rise to these type of integro-

differential equations. Many researchers like K. Balchandran [6], F. E. Browder [7], T. A. Burton [8], J. M. Cushing [9], M. B. Dhakne [10], W. E. Fitzgibbon [11], R. C. Grimmer [12], G. Grinenberg and S. O. Londen [13], J. K. Hale [14], M. L. Heard [15], A. J. Jerri [16], G. E. Ladas and V. Lakshmikantham [17], R. K. Miller [18], B. G. Pachpatte [19], C. C. Travis and G. F. Webb [20] etc. have made numerous and substantial contributions to the theory of integral and integro differential equations.

2) Review of Literature and Development in the subject (Previous work done in the relevant areas):

Delay integro differential equations in recent years have been an object of investigation with increasing interest. For more information see the monographs, K. Gopalsamy et.al, and V. B. Kolmanovskii and A. Myshkis et.al. The research papers cited therein. Also Many authors has studied existence ,uniqueness and other properties of delay integro differential equations of more general type or their special forms such as C. Tunç (2018), A. Verdugo (2018), F.A. Khasawneh (2011), R.P. Agarwal (2011), B IRAGI (2017), F.A. Rihan (2009) , M. Kudu (2016), S. Anwar (2015), K. Ezzinbi (2016),Y.Yang (2013), I. Komashynska (2016), X .Zhou(2017), P. Stechlinski (2014), and H.R. Henríquez (2007) etc.

3) Objective of Research/ proposed hypothesis:

The purpose of this study is to investigate series solution of various types of linear and nonlinear delay integro-differential equations .Also we study existence, uniqueness, boundedness, stability etc.of different delay integro-differential equations.

4) Methodology to be adopted:

We use different techniques to obtain the results such as:

1) Fixed point theorems.

2) Method of upper and lower solutions.

3) Monotone iterative methods.

4) Numerical techniques.

5) Semi group theory etc.

5) Importance of Study/ society applications:

- **International Status:**

Many mathematicians are working in this field, some of them are Agarwal (Singapore), Myshkis A.D (Russia), Alex Verdugo (Mexican-American), Fathalla A. Rihan (United Arab Emirates), Oklahoma (United States of America), C.T.H .Baker (United States of America),Michel Grabisch(Subotica, Serbia),C.Zhang (China)),R.P. Agarwal (Missouri, USA) Lianzhong. Li (China), N. I. Mahmudov, (Turkey), etc.

- **National Status:**

Some of the Indian Mathematicians are also working on same line. Few of them are A.K.Srivastava, (Varanasi), R.Sakthivel (Coimbatore, TamilNadu),M.B.Dhakne(Aurangabad,Maharashtra),M.MallikaArjunan (Coimbatore,TamilNadu),SelvarajSuganya(Coimbatore,TamilNadu)R.Ganesh(Coimbatore,TamilNadu),R.S.M.Anthoni(Coimbatore ,TamilNadu) ,B.G.Pachpatte(Maharashtra),K.Balachandran(Coimbatore ,Tamil Nadu), S. Abbas (Mandi), D. Bahuguna (Kanpur), D.N. Pandey (Roorkee) etc.

- **Significance of study:**

Many problems of partial differential and integro-partial differential equations and application of integro differential equation play a major role in the mathematical modeling of many fields such as physical, biological phenomena and engineering sciences that necessarily take into account the effect of real world problems.

6) Proposed Work plan /Formulation and Structure of study:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic, like delay integrodiff eqn , some fixed point theorems etc.	Good command on the subject and clear the course work.
2	Second year	Collecting the papers and analyze them.	Try to get some new result based on the previous work.
3	Third Year	Work on some problems to be included in the thesis.	Submit at least two papers.
4	Fourth Year	Publishing some papers and submitting the Thesis.	Some more papers and Thesis submission.

7) References:

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